

Digital Gamma Approach to Calculating Margin

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This document proposes an approach for computing the initial retained collateral for bitcoin. The purpose of the initial retained collateral is to protect our customers against further losses after a liquidation decision has been made due to losses from detrimental price movements, BUT BEFORE the liquidation has been fully executed to its completion.

The goal of our process is to both put a number on what should, under most circumstances, cover a default and also to provide guidance to customers about the business risk they are taking both with and without using TPR and our recommendations. To do that, we use as raw material:

- 1) Historical data analysis of the daily price movements of bitcoin in terms of percentiles
- 2) Parametric probabilistic analysis by fitting probability models to the moments (mean, standard deviation and higher moments) obtained from historical data, and using the resulting probabilistic models to estimate the extent of extreme daily moves in terms of percentiles

Historical Data Analysis

For historical data analysis, we first need to obtain the historical data of daily bitcoin prices. There are various publicly available sources, an example of which is Yahoo on this webpage: <https://finance.yahoo.com/quote/BTC-USD/history?p=BTC-USD>.

The data from the time period before Jan 2015 is not used because:

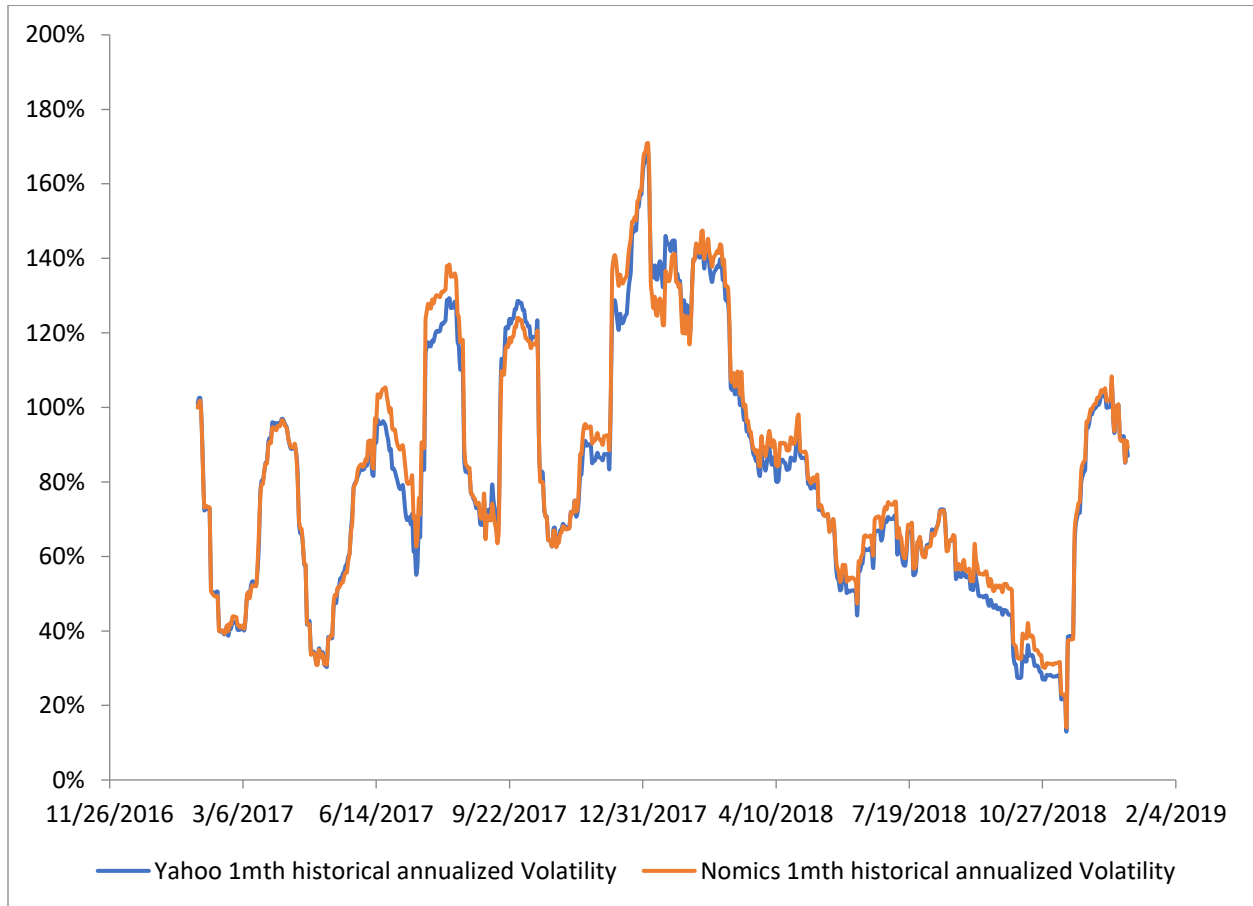
- 1) There is insufficient depth to bitcoin trading prior to Jan 2015
- 2) The percentage moves are inaccurately exacerbated due to the low underlying spot price before Jan 2015

Yahoo Finance obtained the historical data of cryptocurrencies from CoinMarketCap (CMC)¹. It is known that CMC incorporates data from various exchanges, even when arbitrage opportunities exist across exchanges, especially international ones. One such example is the so-called “kimchi premium”, where the prices of bitcoin listed on Korean exchanges are persistently higher than other exchanges in 2017-2018, such that one can potentially obtain a risk-free profit by buying on other exchanges, and simultaneously selling it on the Korean exchange. As such, we want to verify the impact of such price discrepancies on the calculation of historical volatility compared to another source.

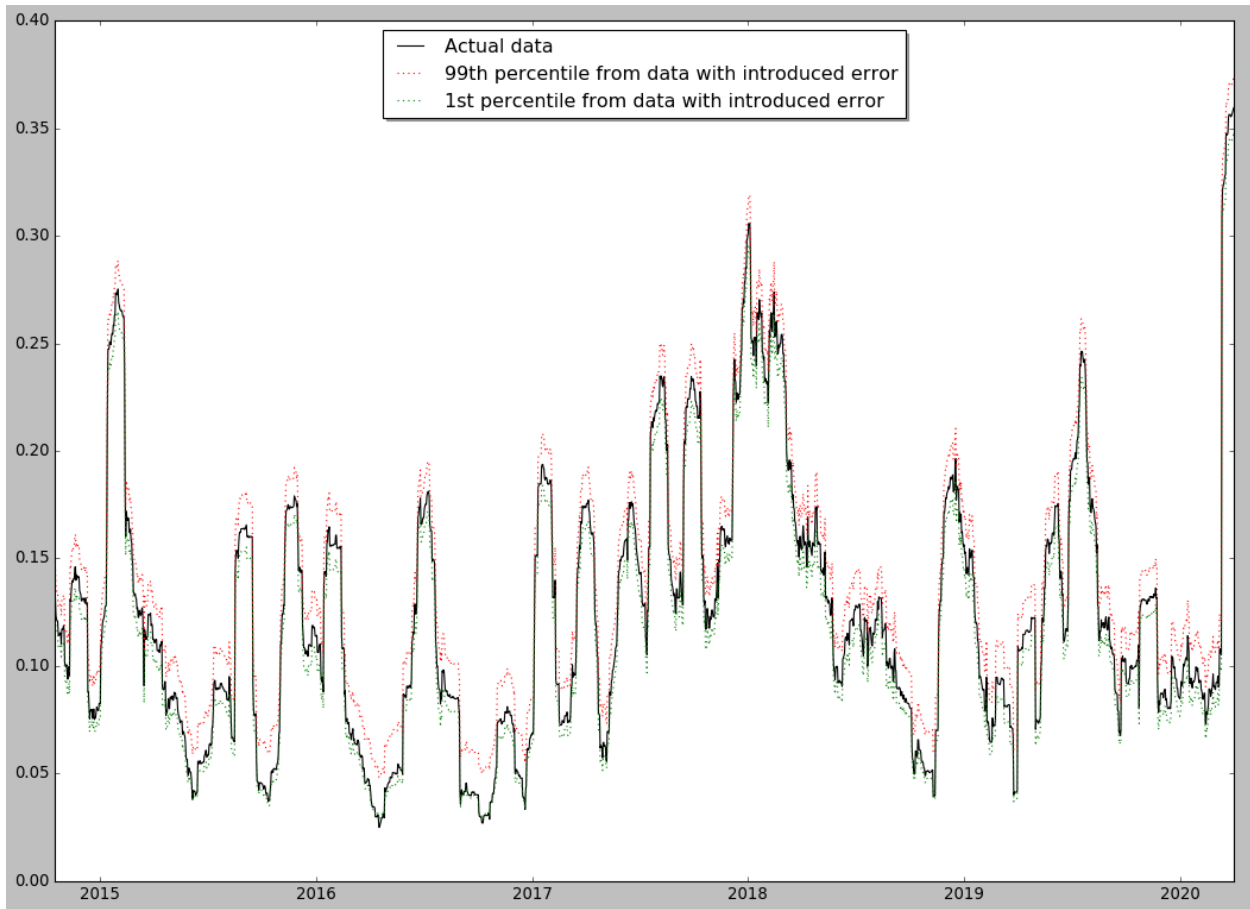
In this case, we used the historical data of bitcoin from Jan 1, 2017 to Dec 31, 2018 from Yahoo Finance, and compared it to another alternative source, Nomics, which is a company focused on providing high-quality cryptocurrency data. We used the daily historical data from both sources and computed 1 month

¹ <https://coingecko.com/news/yahoo-finance-adds-coinmarketcaps-crypto-prices-to-its-website>

rolling historical volatility. The chart below indicates that the difference in the results is not noticeably different, which gives us comfort in using the data from Yahoo Finance despite their shortcomings.



Also, we want to confirm the quality of the data from Yahoo Finance (and CMC) and the impact on our analysis due to potential inconsistencies and errors in the data-collection process. To do so, for the daily data of Bitcoin obtained from Yahoo Finance from Jan 1 2015 to Mar 24 2020, we add a random error term to the price on each day (we used a uniform random number between -1% and 1%, which we think should serve as a conservative representation of the error), compute the 1 month-historical volatility as before, and repeat the process 10,000 times (or simulations). From these 10,000 realizations, we can obtain the 99th percentile and 1st percentile of the rolling 1 month historical annualized volatility, and compare it to the same data without any error introduced. The chart below shows the time-history of these 3 separate cases, and it can be seen that the difference is not noticeably different. This also provides us with more comfort in using the data that we obtained from Yahoo Finance.



Since we are now comfortable with using Yahoo Finance as the source of our data, here are the statistics in various percentiles of 1 day historical daily moves from Jan 1 2015 to Mar 24 2020. The 99th percentile based on the historical data analysis starting from Jan 2015 is about 11.7%. As alluded to earlier, the base assumption is that we will be able to liquidate, in full, the position over a period of one day; we can certainly consider longer time periods for larger positions, or if liquidity is deemed to be insufficient.

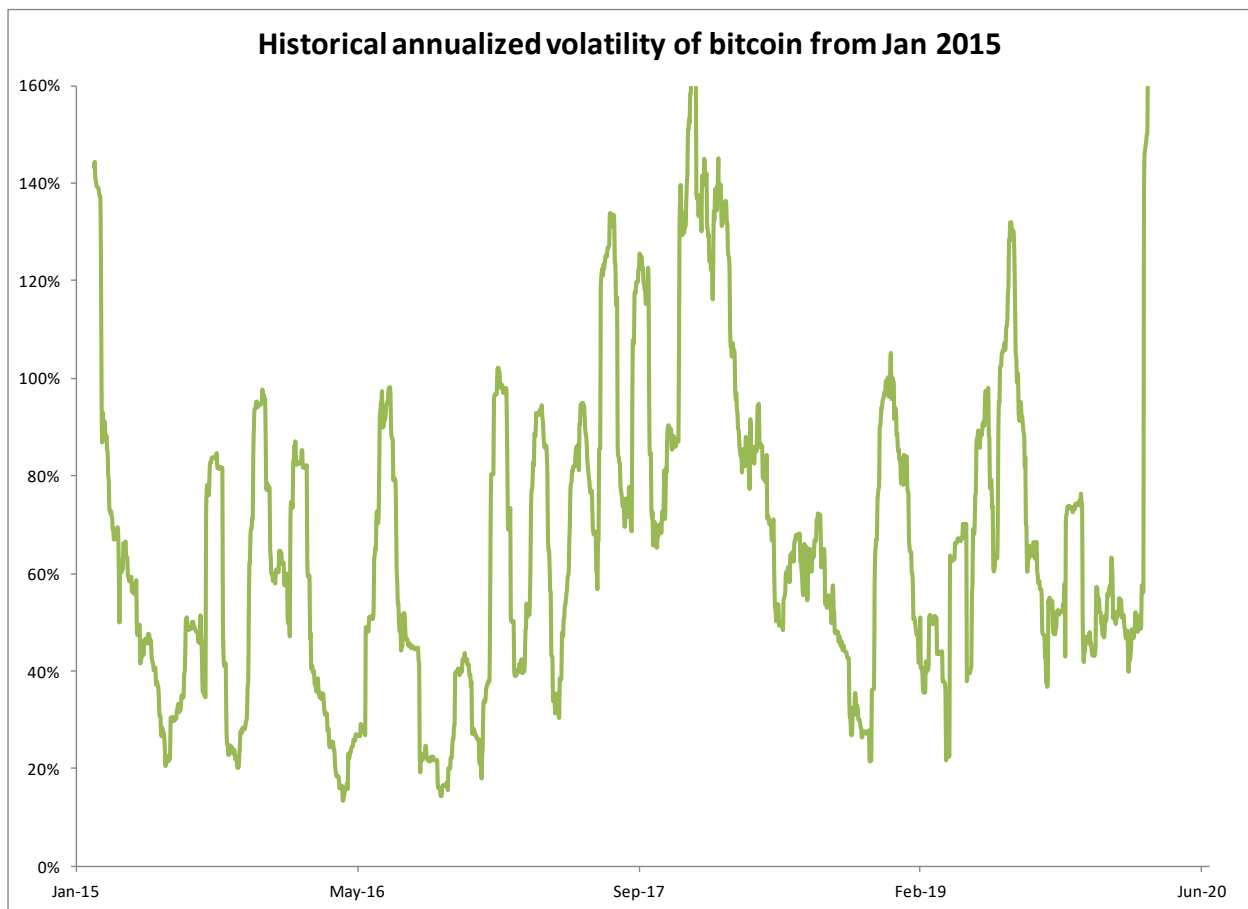
75th percentile of historical daily move from Jan 1 2015	1.8%
95th percentile of historical daily move from Jan 1 2015	6.3%
99th percentile of historical daily move from Jan 1 2015	11.7%

Parametric Probabilistic Analysis

The purpose of parametric probabilistic analysis is to generalize from the available historical data a suitable probability distribution function of the daily price changes of bitcoin. Similar to the approach that is typically utilized for pricing equity derivatives, we consider 2 probability models: a lognormal model and a student-t model.

Both models require as a basic input the historical volatility of bitcoin returns. This volatility can be calculated as the standard deviation of the daily one-day returns using the available data from Jan 2015, and subsequently converting to an annualized basis by multiplying by the square root of 365.

Here is a chart of rolling 1m annualized historical volatility of bitcoin using the data from Jan 2015.



A common approach to model price changes (especially in the equity derivative world) is to utilize a lognormal approach, where the daily price changes are assumed to be normal (or Gaussian). A fatter-tailed model (as compared to lognormality) is the student-t distribution with 10 degrees of freedom (as the degree of freedom increases from 10 to 30, the student-t distribution gradually approaches a normal distribution, i.e., a student t-distribution with 10 degrees of freedom has a “fatter” tail than a student t-distribution with 30 degrees of freedom or a normal distribution). We use the mean and standard

deviation (or historical volatility) of the daily returns from historical analysis, and fit the 2 parameters to both the lognormal and student-t distribution. We can then use the resulting probability models to estimate the various percentiles of daily returns. Note in place of the historical volatility, we can potentially also use the implied volatility of options traded on various exchanges (Deribit, for example) for our analysis; we can also stress the implied volatility for calculations of derivative products, similar to what CME does for their futures options

Here is a table summarizing the daily percentage changes at 95th and 99th percentiles using both the lognormal model and the student-t model, and comparing them with the results of the historical data analysis.

Percentile	Historical Data Analysis	Lognormal	Student-t
95%	6.3%	6.9%	24.9%
99%	11.7%	7.9%	33.8%

Note that the 33.8% number obtained from the fat-tailed student-t distribution is generally in-line with CME initial margin for their futures (~35%).